Wow: 690 students enrolled !

PHY 117 HS2023

Week 3, Lecture 1

Oct. 3rd, 2023

Prof. Ben Kilminster

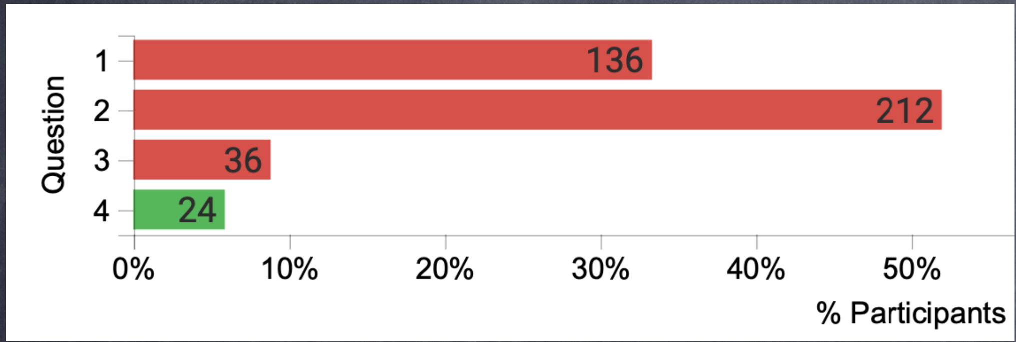
Please do quiz #2 on OLAT

Week 1 online OLAT quiz

Participants	408

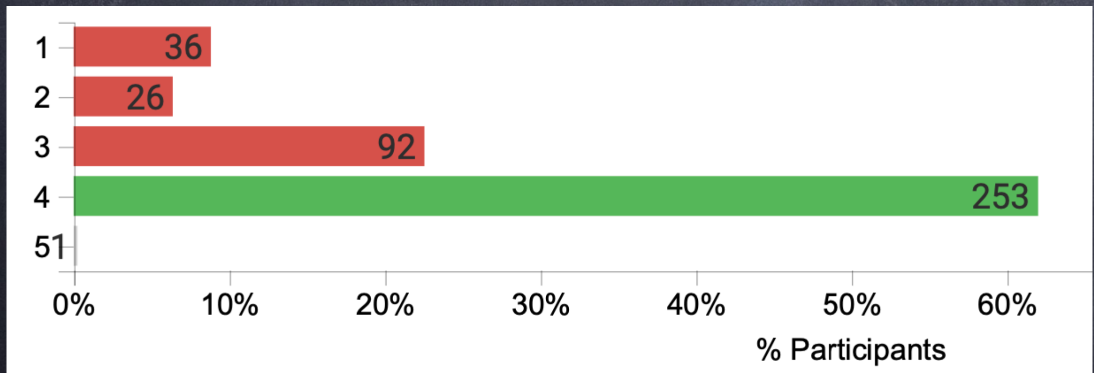
60% participation

Who are you ?



biology
bio-medicine
bio-diversity
other

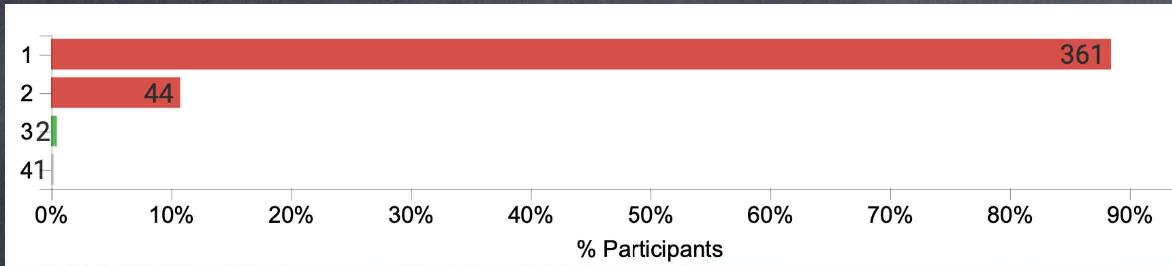
How many semesters of physics have you had ?



0
1
2
3+

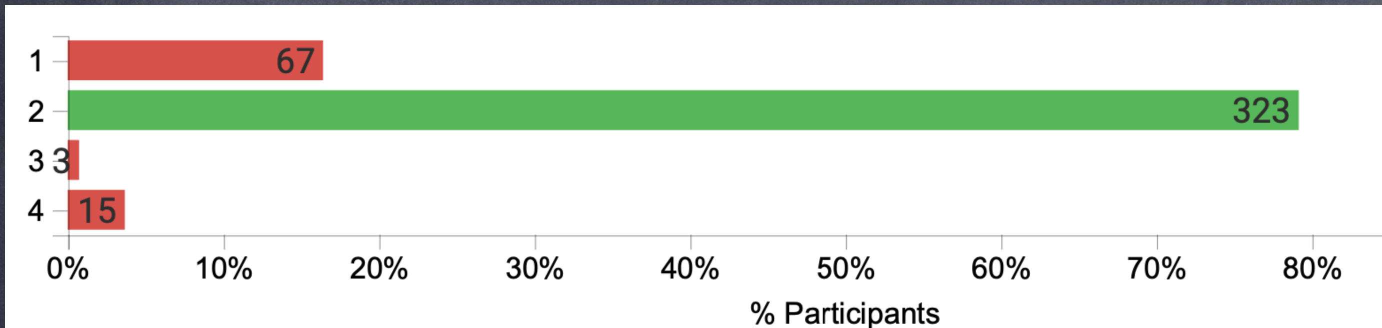
9% no physics
62% 3+ semesters

Are you taking MAT 182 ?



Yes
No
Undecided

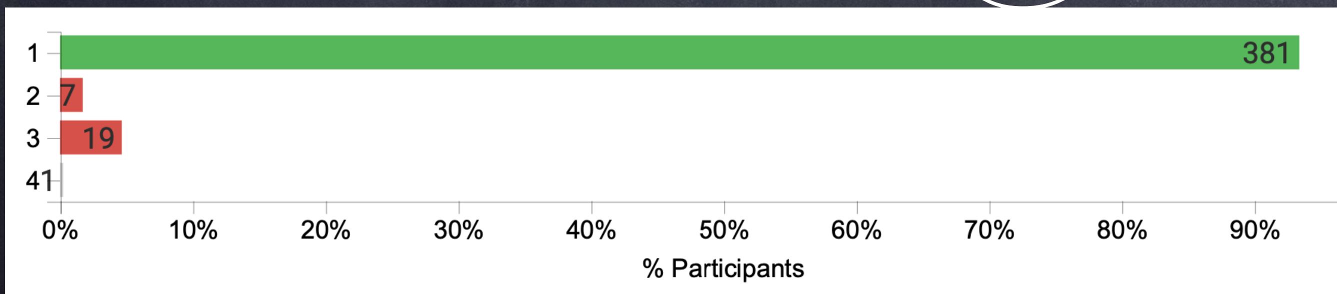
What could be the integral of $3\,x^2$?



- $6\,x$
- x^3
- 0
- What's an integral ?

What is the derivative of x^3 with respect to x ?

$3x^2$



- $3x$
- $3x^2$
- $2x^2$

Unanswered	Right	Wrong	
		★	A unit vector has a magnitude of zero.
		★	if a particle is moving at a constant velocity, the slope of distance vs. time will be zero.
	★		the position of a simple harmonic oscillator repeats in a time of $\frac{2\pi}{\omega}$.
	★		On the moon, a metal ball and a feather thrown from one astronaut to another would have the same parabolic motion.
		★	The acceleration of an object moving in a circle points in the same direction as the velocity.

Questions all at least 70% correct

	Unanswered	Right	Wrong
A unit vector has a magnitude of zero.	14	40	354
if a particle is moving at a constant velocity, the slope of distance vs. time will be zero.	13	113	282
the position of a simple harmonic oscillator repeats in a time of $\frac{2\pi}{\omega}$.	62	274	72
On the moon, a metal ball and a feather thrown from one astronaut to another would have the same parabolic motion.	13	282	113
The acceleration of an object moving in a circle points in the same direction as the velocity.	15		291

Types of energy:

- work
- kinetic energy
- potential energy $\nearrow$ due to gravity, $\searrow$ due to spring

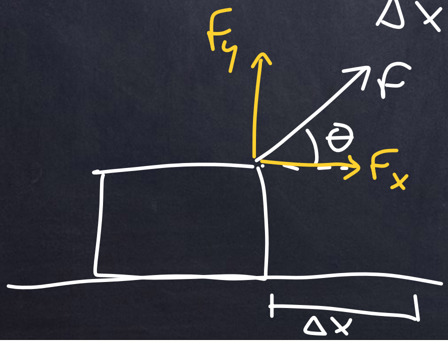
Relationship of forces to energy:

$$\underset{\text{force}}{[N]} = \left[\frac{kg\,m}{s^2}\right] \qquad \underset{\substack{\text{energy}\\ [\text{Joule}]}}{[J]} = \left[\frac{kg\,m^2}{s^2}\right] = \underset{\text{force}\,\cdot\,\text{distance}}{[N \cdot m]}$$

The work done by a force is $W = F\Delta X$

(for the case when $\bar{F}$ is in the direction of $\bar{F}$)

$$W = F\Delta X$$



If $\bar{F}$ not parallel to $\overline{\Delta X}$, we need to find the component of $\bar{F}$ that is parallel.

$$W = F_x \Delta X = F\cos\theta\,\Delta X$$

(so $F_x \parallel \Delta X$)

When the force is in the same direction as the motion, W is (+)

Derivation: If we have a net force, then we get an acceleration

$$\sum F_x = ma$$

Remember $v^2 = v_0^2 + 2a\Delta x \Rightarrow a = \dfrac{v^2 - v_0^2}{2\Delta x}$

$$\text{work} = F_x \Delta x = ma\Delta x = m\left(\frac{v^2 - v_0^2}{2\cancel{\Delta x}}\right)\cancel{\Delta x} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

The work-energy theorem:

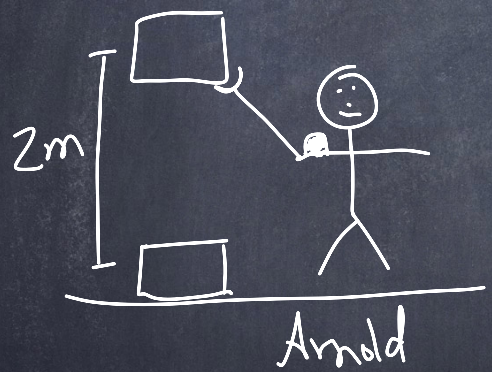
$$\boxed{W_{TOTAL} = \frac{1}{2}m\underset{\substack{\uparrow \\ \text{final}}}{v_f^2} - \frac{1}{2}m\underset{\substack{\uparrow \\ \text{initial}}}{v_i^2} = K_f - K_i = \Delta K}$$

$$\boxed{\frac{1}{2}mv^2 \equiv \text{Kinetic energy} = K}$$

is the kinetic energy of an object moving

Notes:
- K is a scalar, no direction
- K is always positive or zero
- ΔK can be negative (if object slows down)
- Consider each force separately, and the work it does.

Example:



Arnold lifts a 5 kg block to $h = 2m$, using 500 N of force.

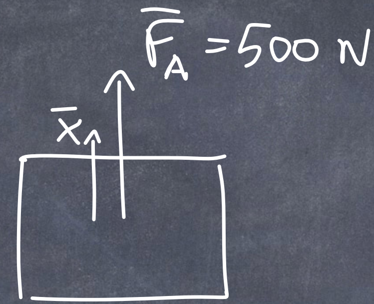
1) what is the work done by Arnold?
2) what is the work done by gravity?
3) what is the final velocity of the block?

There are 2 forces at work, Arnold + gravity.
Initial velocity is zero.

work done by Arnold.

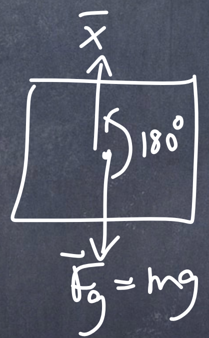
+X

1)



$$W_A = F_A \cancel{\cos\theta}^{\,1}\, \Delta X = F_A \Delta X$$
$$= (500\,N)(2m)(1)$$
$$= 1000\ J$$

2)



$\theta = 180°$

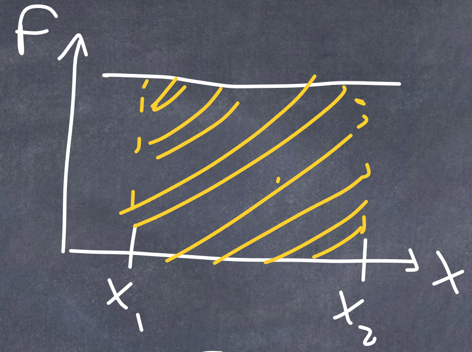
$$W_g = F_g \cancel{\cos\theta}^{\,-1}\, \Delta X = -mg\Delta x$$
$$= -(5\,kg)\left(10\,\frac{m}{s^2}\right)(2m)$$
$$= -100\ J$$

3) $W_{TOTAL} = W_A + W_g = 1000\,J - 100\,J = 900\,J$

$$W_{TOTAL} = \Delta K = \frac{1}{2}mv_F^2 - \cancel{\frac{1}{2}mv_i^2}^{\,0}$$

$$v_F = \sqrt{\frac{2(W_{TOTAL})}{m}} = \sqrt{\frac{2(900\,J)}{5\,kg}} = 19\,\frac{m}{s}$$ up direction

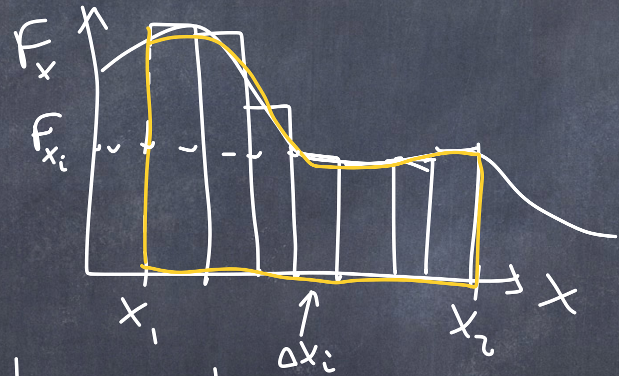
So far, we have had a constant force



we move an object in x-direction.

$W = F\Delta x$ = area under F vs. x curve.

what if our force is changing?



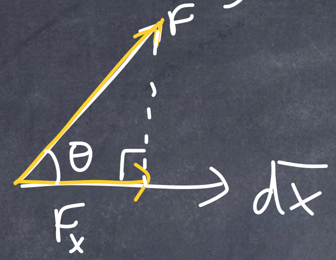
F_x moves only in x direction

The total work is $W = \lim_{\Delta x_i \to 0} \sum_i F_{x_i} \Delta x_i$

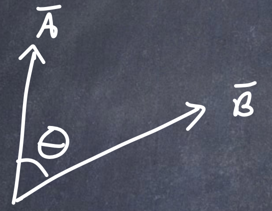
$$\boxed{W = \int_{x_1}^{x_2} F_x \, dx}$$

$F_x \equiv F(x)$ The x-component of the force, in the direction of movement, x.

So far, we have $W = \int F_x \, dx = \int_{x_1}^{x_2} (F\cos\theta)\, dx$



In physics, we often have vectors in different directions



The dot product of $\bar{A}$ and $\bar{B}$ is:

$$\bar{A} \cdot \bar{B} = |\bar{A}|\,|\bar{B}|(\cos\theta)$$

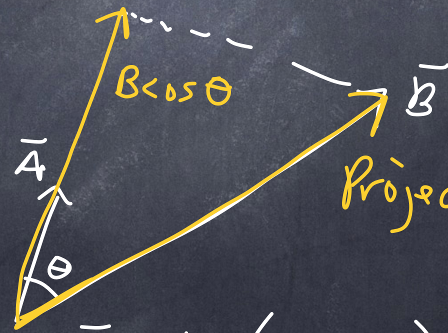
A dot product multiplies the parallel components of 2 vectors.

Two ways:

Projection of $\bar{A}$ onto $\bar{B}$

$$\bar{A} \cdot \bar{B} = (A\cos\theta)(B) = AB\cos\theta$$

where A and B are magnitudes



Project $\bar{B}$ onto $\bar{A}$

$$\bar{A} \cdot \bar{B} = (B\cos\theta)(A)$$
$$= AB\cos\theta$$

In 3-D coordinates (x,y,z)

$$\bar{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$
$$\bar{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

$$\underbrace{\bar{A}\cdot\bar{B}}_{\text{tells us how parallel are our vectors.}} = A_xB_x + A_yB_y + A_zB_z$$

A scalar value (not vector)

So our formula is now $W = \int_{s_1}^{s_2} \bar{F}\cdot d\bar{s}$

$\bar{s}$ is the path of the object.

$d\bar{s}$ could be $d\bar{x}$ if the object is moving in the x-direction.

In this class, we will only deal with paths that are in one direction at a time.

The work done on a system can be <u>stored</u> as potential energy, U.

Potential energy can change $\Delta U = U_f - U_i = U_2 - U_1$

Sometimes, we write $dU = \Delta U$

$$\boxed{-\Delta U = -(U_2 - U_1) = W = \int_{s_1}^{s_2} \bar{F} \cdot d\bar{s}}$$

Notice that $\boxed{\Delta U = -W}$

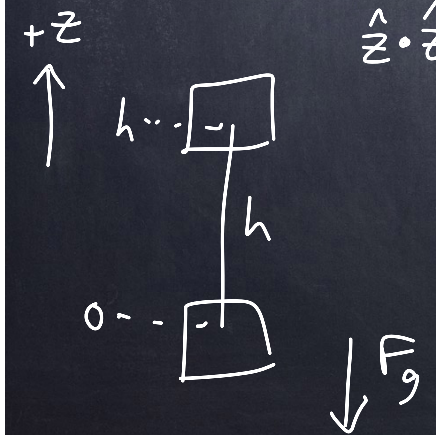
Example:

Assume we lift an object of mass, m to a height, h.

$\hat{z} \cdot \hat{z} = 1$

Force of gravity is $F_g = mg$, $\bar{F}_g = -mg\hat{z}$

$d\bar{s} = dz\,\hat{z}$ $\quad W = \int \bar{F} \cdot d\bar{s}$



work done by gravity $\quad W = \int_0^h (-mg\,\hat{z}) \cdot (dz\,\hat{z}) = \int_0^h -mg\,dz$

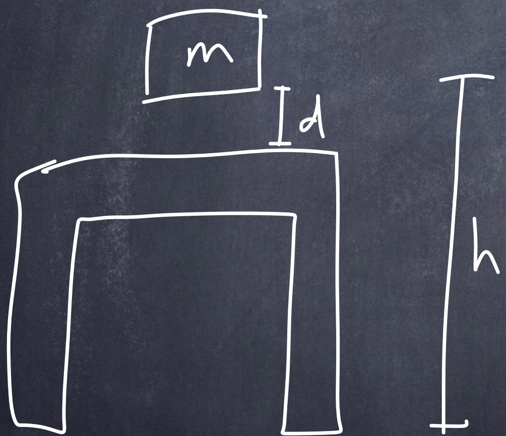
$W = -mg\Big]_0^h = -mgh$

change in potential energy $\quad \Delta U = -W = -(-mgh) = mgh$

In general, gravitational potential energy is $U = mgz$

$\uparrow$ height

But potential energy is relative.



$U = mgz$

U relative to the floor is $U = mgh$

U relative to the table is $U = mgd$

Conservation of energy

$$\boxed{E_{before} = E_{after}}$$

always true.
If we consider all sources of energy, then the sum of energies is conserved before and after any situation.

For conservative forces, potential energy plus kinetic energy is conserved.

$$\boxed{(K + U)_{before} = (K + U)_{after}}$$

summary so far,

Energy is conserved $\boxed{E_{before} = E_{after}}$

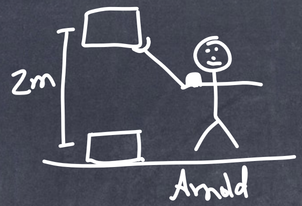
$\boxed{\text{Potential energy}_{\text{gravitational}} = mgh = U}$

relative to a height of zero

$\boxed{\text{Kinetic energy} = \frac{1}{2}mv^2 = K}$

work-energy theorem $\boxed{W_{TOTAL} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$

potential energy & work relation $\boxed{-\Delta U = W = \int_{s_1}^{s_2} \bar{F} \cdot d\bar{s}}$



Consider Arnold again.

If he lifts the block with 50N of force, to $h = 2m$ and lets go, how fast will it move as it hits the ground.

1) work of Arnold : $\vec{F}_A \uparrow \quad \uparrow \vec{\Delta x}$

$$W_A = \vec{F}_A \cdot \vec{\Delta x} = \underbrace{F_A \Delta x}_{\cos\theta = 1} = F_A h$$

$$= (50\,N)(2m) = +100J$$

2) gravity also does work

$\vec{x} \uparrow$ $\theta = 180°$ $\downarrow \vec{F}_g$ $\cos 180° = -1$

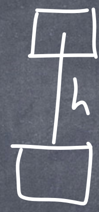
$$W_g = \vec{F}_g \cdot \vec{\Delta x} = -F_g \Delta x = -mgh = (-5kg)\left(10\frac{m}{s^2}\right)(2m)$$

$$= -100\,J$$

3) Total work $= W_A + W_g = 100J - 100J = 0\,J$

The total work is zero, so velocity is zero.

Arnold drops the weight, how fast is it moving?



The block has $\left.\begin{array}{l} U = mgh \\ K = 0 \end{array}\right\}$ at top

When he drops it, the potential energy becomes kinetic energy.

$$(K + U)_{\text{Before}} = (K + U)_{\text{after}}$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

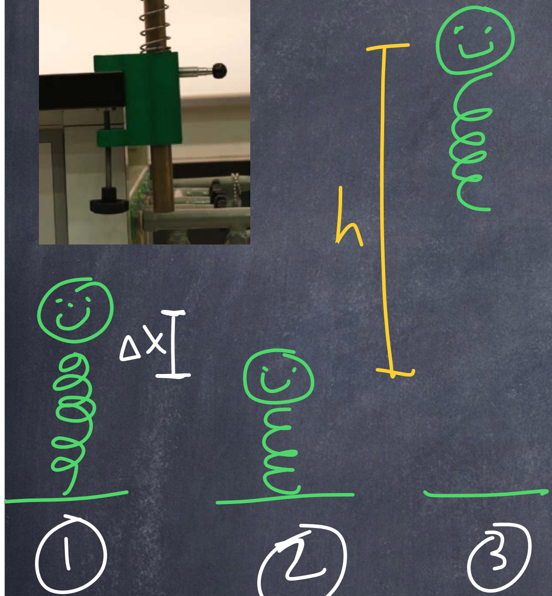
$$V = \sqrt{2gh} = \sqrt{2 \cdot \left(10\frac{m}{s^2}\right) 2m} = \sqrt{40}\ \frac{m}{s}$$

$$= 6.3\ \frac{m}{s}$$

How high does the grasshopper jump?

$F_s = -k\Delta X$

force points opposite the stretching of the spring.



$$K = \frac{F_s}{\Delta X} = \frac{(2.5\,kg)(9.81\,m/s^2)}{0.04\,m} = 612.5\ \frac{N}{m}$$

$$-\Delta U = W = \int \bar{F} \cdot d\bar{s}$$

$$W = \int_0^x F \cdot dx = \int_0^x (-kx) \cdot dx$$

$$W = -\frac{1}{2}kx^2$$

$$\Delta U = -W = \frac{1}{2}kx^2$$

$$\boxed{U = \frac{1}{2}kx^2}$$

energy stored in a spring

where x is the compression of spring