PHY117 HS2023

Week 6, Lecture 1
Oct. 24th, 2023
Prof. Ben Kilminster

week 5 quiz coming soon!

To avoid confusion, we have 4 ks:

K = Kelvin

k = Boltzmann constant

K = kinetic energy

κ = coefficient of thermal conductivity

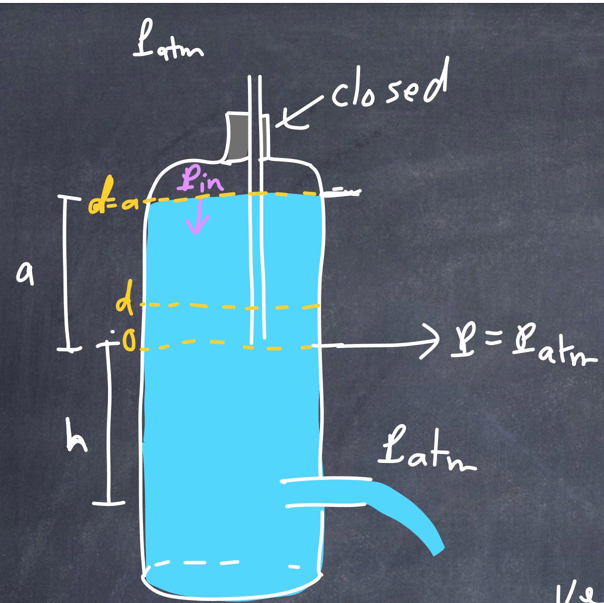
Thermodynamics – study of temperature, heat, and the exchange of energy. (mechanical work)

Macroscopic scale: measurable properties:

volume, pressure, temperature,

pressure → force/area

Marriotte's bottle:



The P_{atm} is transmitted to the depth marked as "a". P_{in} must be $P_{atm} - \rho g d$.

The velocity will be $v = \sqrt{2gh}$

The benefit here is that the velocity of water coming out does not change within the range of a.

How does this work?
P_{in}, pressure at the top) will change.

The P_{in} will be negative at the start, because this is a difference to P_{atm}

$$\underset{\uparrow \text{top}}{P_{in}} + \rho g d = \underset{\uparrow \text{bottom}}{P_{atm}} \quad (\text{for } 0 < d < a)$$

$$P_{in} = P_{atm} - \rho g d$$

when $d=0$, $P_{in} = P_{atm}$
when $d=a$, $P_{in} = P_{atm} - \rho g a$

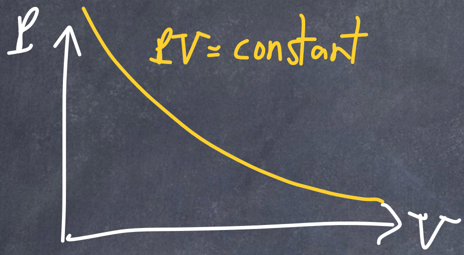
If $d = 10$ cm, then:

$$\rho g d = \frac{1\,g}{cm^3} \cdot \frac{1\,kg}{1000\,g} \left(\frac{100\,cm}{1\,m}\right)^3 \left(10\,\frac{m}{s^2}\right) (10\,cm) \frac{1\,m}{100\,cm} = 1\,kPa$$

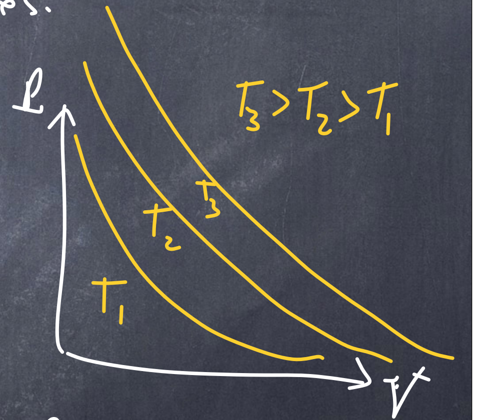
From Marriotte's bottle (fluids)
to Marriotte's Law (gases)

Ideal gases: randomly moving point particles with no inter-particle interactions

Boyle-Marriotte law: $PV =$ constant for an ideal gas at constant temperature.



As we change temperature, PV takes different values.



The law is written as:

$$\boxed{PV = nRT = NkT \quad \text{Ideal gas law}}$$

pressure $[Pa] = [\frac{N}{m^2}]$

volume $[m^3]$

temperature $[K]$

we see that $nR = Nk$

N: number of gas molecules

k: Boltzmann constant, $k = 1.38\,e{-23}\ J/K$

n: number of moles

R: gas constant, $R = 8.314\ J/K\cdot mol$

Note: $R = N_A \cdot k$

$N = n \cdot N_A$

where N_A: Avogadro's number is # molecules/mole

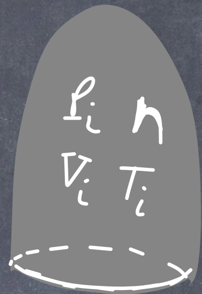
$N_A = 6.02\,e23\ \frac{\text{molecules}}{\text{mole}}$

Why does our hot air balloon rise when air inside is heated?

pressure decreases?

density decreases?

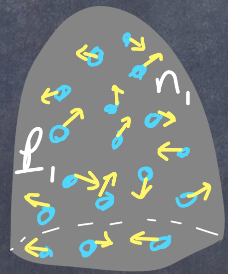
volume increases?

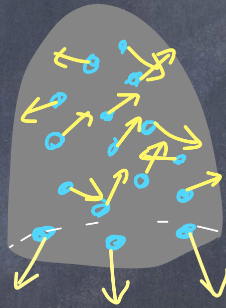


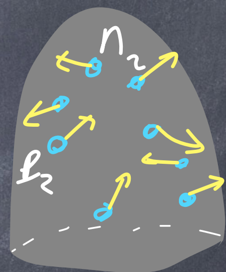
Visually:

before heating | during heating | after heating

P_{atm}







Initially:

P_1 : pressure inside

$P_1 = P_{atm}$

V_1 = volume inside

n_1 = # moles air inside

T_1 = temp. inside

molecules speed up and push out of balloon

Finally:

$P_2 = P_1 = P_{atm}$

$V_2 = V_1$

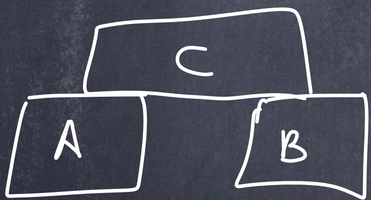
$n_2 < n_1$

$T_2 > T_1$

What is temperature? Intuitively, it is a measure of the hotness or coldness of something.

Objects have thermometric properties: gases expand, as do most solids + liquids with temperature (if allowed to). Electrical resistance changes...

0^{th} law of thermodynamics: If 2 objects are in thermal equilibrium with a third object, then they are in thermal equilibrium with each other.



C is in thermal equilibrium with A + B (no further thermometric change).

C is the same temperature as A + B.

place A + B in contact

A + B must be the same temperature.

$PV = nRT$

$P \sim T$

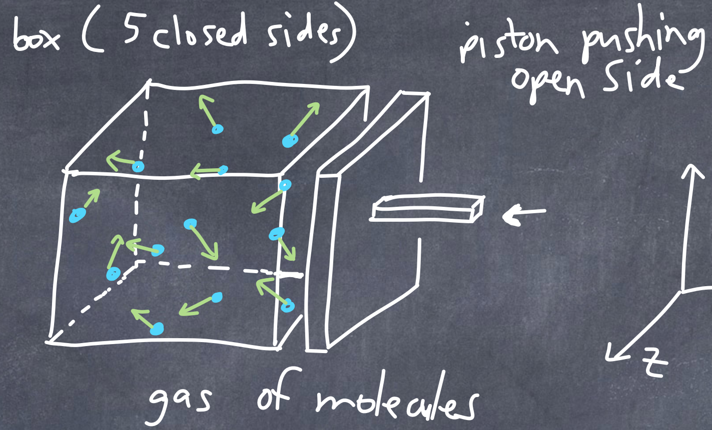
$-273.15°C \hat{=} 0$ Kelvin $\hat{=} 0\,K$

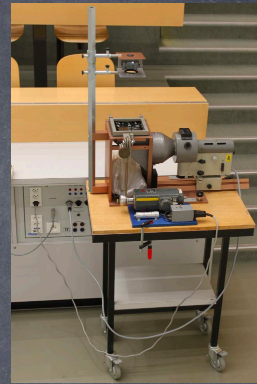
Since 2019, the Kelvin is defined using

$k = 1.380649\,E{-23}\ J/K$

$J = \frac{kg \cdot m^2}{s^2}$ so we can get definitions of the K from m, s, kg

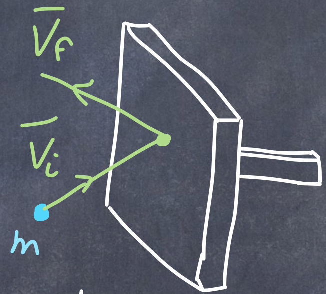
But what is temperature really?
At the molecular level?





we put molecules in a box, and close with a piston.

when one molecule hits the piston, its momentum changes.



initial: $\bar{V}_i = V_{i_x}\hat{x} + V_{i_y}\hat{y} + V_{i_z}\hat{z}$

final: $\bar{V}_f = V_{f_x}\hat{x} + V_{f_y}\hat{y} + V_{f_z}\hat{z}$

considering the x-direction: $\Delta p_x = m V_{f_x} - m V_{i_x}$

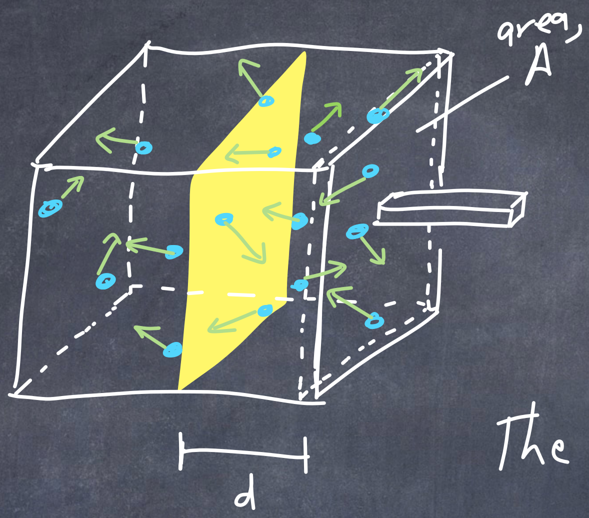
Assume it is an elastic collision, $|\bar{V}_f| = |\bar{V}_i|$

$|V_{f_x}| = |V_{i_x}|$

$\Delta p_x = 2mV_x$ for one molecule in gas

$\left(\Delta \bar{p} = \text{Impulse} = \bar{I}\right)$

total volume V



In some Δt (time), molecules hit the piston. The ones that hit the piston must be less than a certain distance away from the piston, moving to the right, depends on velocity

$$V_x = \frac{d}{\Delta t} \qquad d = V_x \Delta t$$

The volume of this box is $A \cdot d = A \cdot V_x \Delta t$

The number of particles in this box that hit the piston:

$$\underbrace{N_R}_{\text{\# that hit the piston on the right}} = \underbrace{\frac{N}{V}}_{\text{density = \#/volume}} \cdot \underbrace{V_x \Delta t A}_{\text{volume of molecules with correct } V_x \text{ to hit wall}} \cdot \underbrace{\frac{1}{2}}_{\text{moving to the right}}$$

The total momentum of ptcls hitting piston is:

$$\underbrace{\Delta p_x}_{\text{momentum of all molecules}} = \underbrace{\left(\frac{1}{2}\frac{N}{V}v_x \Delta t A\right)}_{\#\text{ molecules}} \underbrace{\left(2mv_x\right)}_{\Delta p \text{ for one molecule}}$$

$$\Delta p_x = \frac{N}{V} m v_x^2 A \Delta t$$

The total force on the wall is $F = \frac{dp}{dt} = \frac{\Delta p}{\Delta t}$

$$F_x = \frac{N}{V} m v_x^2 A \frac{\cancel{\Delta t}}{\cancel{\Delta t}}$$

Pressure is F/A

$$P = \frac{N}{V} m v_x^2 \frac{\cancel{A}}{\cancel{A}}$$

pressure due to v_x of N molecules.

so $PV = Nmv_x^2$

rewrite as $PV = 2N\left(\frac{1}{2}mv_x^2\right) = 2N\left(\begin{array}{c}\text{Kinetic energy} \\ \text{of a molecule} \\ \text{in 1-D}\end{array}\right)$

we recognize that since $PV = NkT$,

then

$$\boxed{\begin{array}{c}\text{Kinetic} \\ \text{energy} \\ \text{of a single} \\ \text{molecule} \\ \text{in 1-D}\end{array} = \frac{1}{2}mv_x^2 = \frac{1}{2}kT}$$

Now we know that temperature is the kinetic energy of the molecules (from their velocity)

$\frac{1}{2}kT$ is the average kinetic energy in 1 dimension

$$K_{1D} = \left\langle \frac{1}{2}mv_x^2 \right\rangle = \frac{1}{2}kT$$

K: kinetic energy

k: Boltzmann constant

Note: $\langle V_x^2 \rangle > 0$ though $\langle V_x \rangle = 0$ since $\frac{1}{2}$ go in $+\hat{x}$ and $\frac{1}{2}$ go in $-\hat{x}$

Also, the molecule has equal velocities in x, y, z directions

So $K_{3D} = \frac{1}{2}m\langle V_x^2 \rangle + \frac{1}{2}m\langle V_y^2 \rangle + \frac{1}{2}m\langle V_z^2 \rangle = 3\left(\frac{1}{2}kT\right)$

Note: $\langle V^2 \rangle = \langle V_x^2 \rangle + \langle V_y^2 \rangle + \langle V_z^2 \rangle = 3\langle V_x^2 \rangle$

So
$$\underbrace{K_{3D}}_{\text{Kinetic energy in 3-D for 1 molecule}} = 3\left(\frac{1}{2}m\langle V_x^2 \rangle\right) = \frac{1}{2}m\langle V^2 \rangle = 3\underbrace{\left(\frac{1}{2}kT\right)}_{\text{Kinetic energy per degree of freedom}}$$

3 degrees of freedom (x, y, z)

we call $\sqrt{\langle V^2 \rangle} \equiv V_{rms}$

V_{rms} = "root mean square" speed

For N particles, the translational kinetic energy (in 3 dimensions) is:

$$K_{3D} = N\left(\frac{3}{2}kT\right) = \frac{3N}{2}m\langle v_x^2\rangle = \frac{N}{2}m\langle v^2\rangle$$

$$\cancel{N}\frac{3}{\cancel{2}}kT = \frac{\cancel{N}}{\cancel{2}}m\langle v^2\rangle \Rightarrow T = \frac{m\langle v^2\rangle}{3k}$$

for 3 degrees of freedom (independent of N)

$$\langle v^2\rangle = \frac{3kT}{m}$$

Q: What is the rms speed of Nitrogen (N_2) molecules at room temperature?

$T = 293\text{ K}$, molar mass $M = 28\,\frac{g}{mol}$

m = mass of 1 molecule $= \frac{M}{N_A}$ $\qquad M = mN_A$

A: $$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\left(\frac{3N_AkT}{mN_A}\right)} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(8.314\,\text{J}/\cancel{\text{K}\cdot\text{mol}}\right)(293\,\cancel{\text{K}})}{0.028\,\text{kg}/\cancel{\text{mol}}}}$$

$$v_{rms} = 510\,\frac{m}{s}$$

Note: velocity depends on T + mass

velocity of particles in a gas:

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}$$

The distribution of velocities is

$$\boxed{f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}}$$

Maxwell-Boltzmann distribution for velocities.

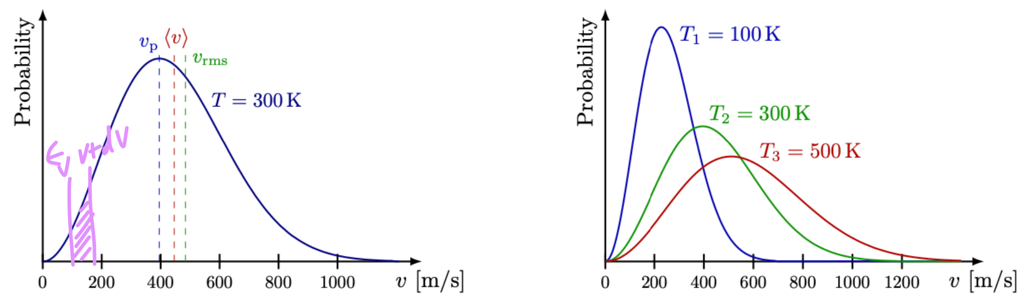
m: mass of a molecule
v: velocity of a molecule
T: temperature

If we had N molecules,

$$dN = N f(v) dv$$

: dN is the number of molecules within a velocity range of $v \to v + dv$

$$\int dN = N = \int_{v_1}^{v_2} N f(v) dv$$



(a) The distribution is asymmetric. **(b)** For different temperatures.

Figure 17.4: Maxwell-Boltzmann distribution for oxygen gas O_2 with atomic weight ~ 16 per atom.

$V_p = V_{max}$: speed for which $f(v)$ is maximum $= \sqrt{\frac{2kT}{m}}$

V_{av} : mean of $f(v)$

V_{rms} : $\sqrt{\langle V^2 \rangle}$

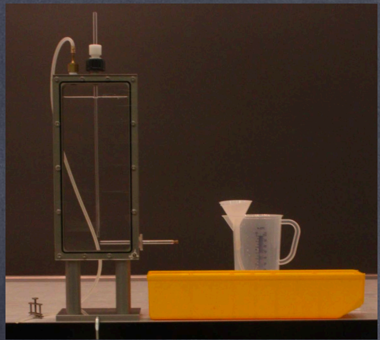
we could rewrite $f(v)$ as $f(E)$ where $E = \frac{1}{2} m v^2$ is kinetic energy

$$\boxed{f(E) = \frac{2}{\sqrt{\pi}} \left(\frac{1}{kT}\right)^{3/2} E^{1/2} e^{-E/kT}}$$

Maxwell-Boltzmann energy distribution.

$$dN = N f(E)\, dE$$

is the # of particles with an energy between E and $E + dE$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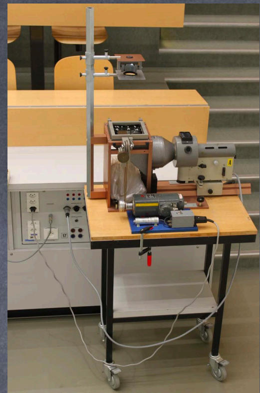
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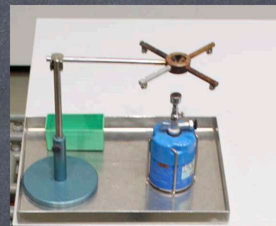
Th12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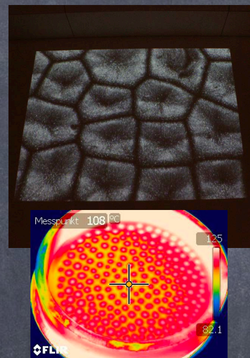
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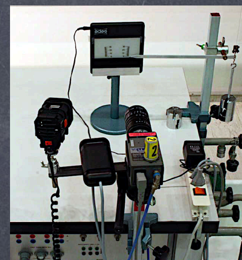
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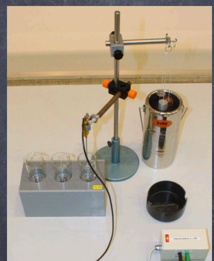
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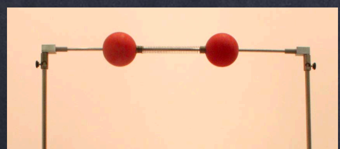
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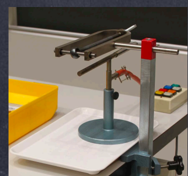
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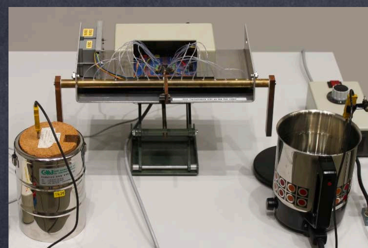
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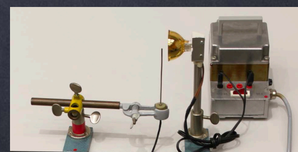
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