PHY117 HS2023

Week 12, Lecture 1
Dec. 5th, 2023
Prof. Ben Kilminster

Last time, standing waves:
in general

$$y(x,t) = 2A\cos\omega t \sin kx$$

where $k_n = \frac{n\pi}{L}$ + $\omega_n = k_n v$ + $v = \frac{\omega}{k}$

$\omega_n = 2\pi n f_1$

$$f_1 = \frac{v}{\lambda_1} = \frac{k_1 v}{2\pi}$$

Example:
standing wave on string

$n=4$

$k_4 = \frac{4\pi}{L}$, $\omega_4 = k_4 v$, $v = \sqrt{\frac{T}{\mu}}$

Vibrating systems have (in general) multiple standing waves, superimposed.

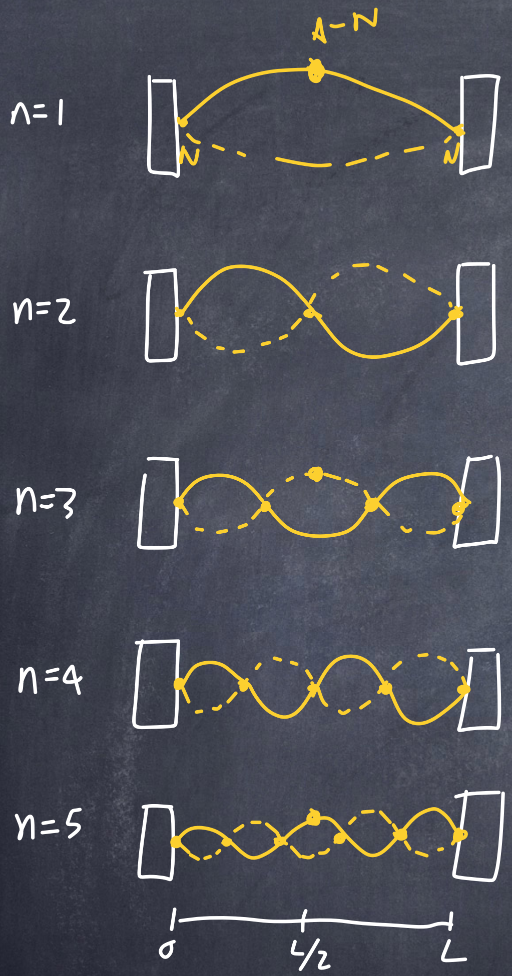
In general $$\boxed{y(x,t) = \sum_n A_n \cos(\omega_n t + \delta_n)(\sin k_n x)}$$

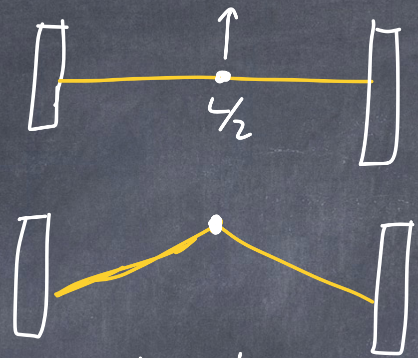
ω_n, k_n : angular frequency, wave number for some n (harmonic value, integer)

A_n, δ_n : Amplitude & phase constants

A_n^2: fraction energy of the n^{th} harmonic in our wave

δ_n : depends on initial conditions

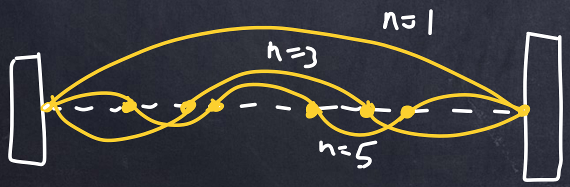




we pluck the string at $\frac{L}{2}$

we excite harmonics, depending on where we pluck the string, to get different A_n.
(relative energies of each harmonic)

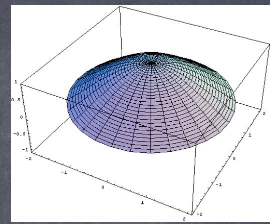
For $\frac{L}{2}$, we create an anti-node in the middle, and this excite the odd harmonics $n = 1, 3, 5, \ldots$

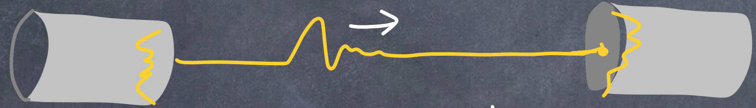


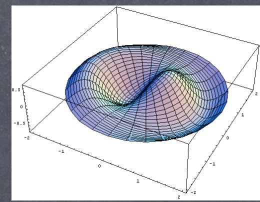
Most energy goes into the fundamental frequency, n = 1

Standing waves on a flat, round surface: diaphragm

T: tension

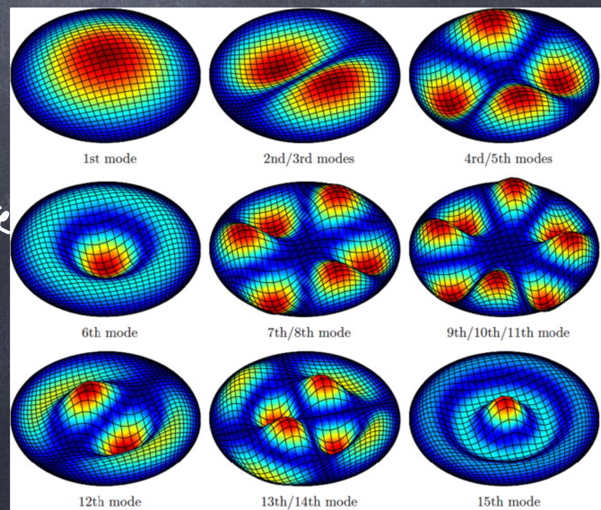


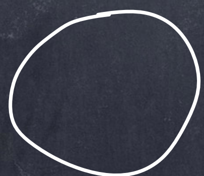




The shape of the harmonics depend on the boundary conditions, which parts are moving + which parts are fixed.

Bessel Functions





The circumference is fixed as a node

Applications of standing waves (PHY 127)

Q: why do electrons in an atom only exist in some states?

A: standing waves

Wavelengths for Different States

For a hydrogen atom:

Electron wave resonance

$n = 1$

$\lambda_1 = 2\pi r_1 = 6.28 a_0$

$n = 2$

$2\lambda_2 = 2\pi r_2$

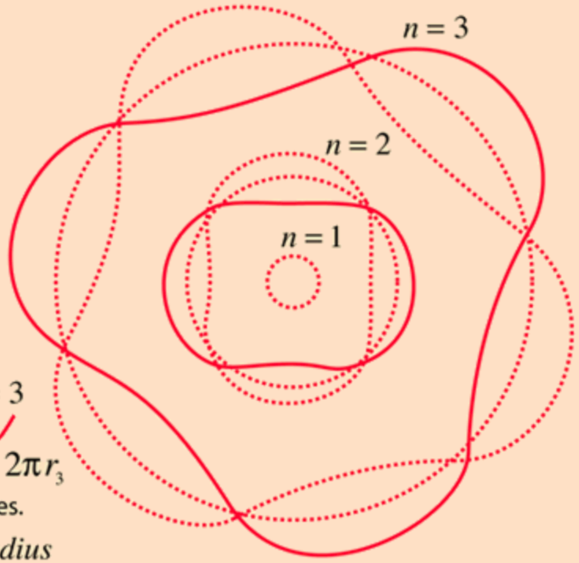
$\lambda_2 = 12.57 a_0$

$n = 3$

$\lambda_3 = 18.85 a_0$ $\quad$ $3\lambda_3 = 2\pi r_3$

Wavelengths for hydrogen states.

$a_0 = 0.0529 nm = Bohr\ radius$



Bohr model of the atom

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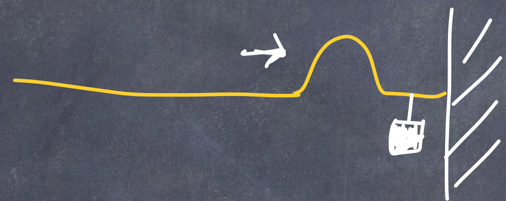
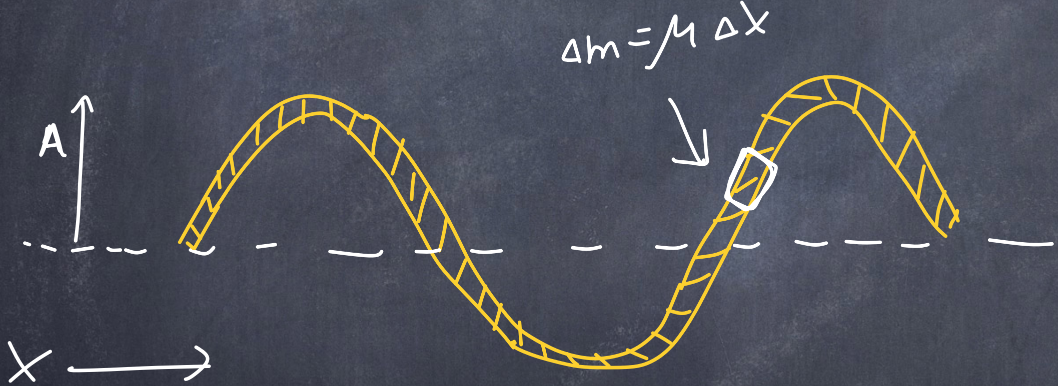
Bohr model concepts

Energy transmission in a wave (on a string)

wave can do work



pulse lifts the weight



we have a sine wave on a string, with amplitude, A, and angular frequency, ω. It has a $\frac{\text{mass}}{\text{length}} = \mu$.

Our piece of string (Δx) oscillates up & down as a simple harmonic oscillator (s.h.o.)

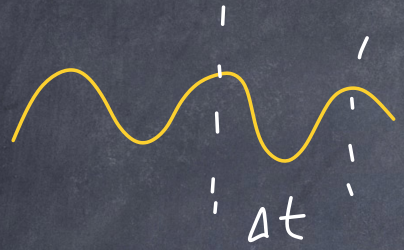
Previously, we determined that the energy of a s.h.o.

k: spring constant

is $E = \frac{1}{2} k A^2 \qquad \omega = \sqrt{\frac{k}{m}} \qquad k = \omega^2 m$

$$\Delta E = \frac{1}{2} k A^2 = \frac{1}{2}\omega^2 \Delta m A^2 = \frac{1}{2}\omega^2 \underbrace{(\mu \Delta x)}_{(\Delta m)} A^2$$

$$\text{Energy} = \Delta E = \frac{1}{2}\omega^2 \mu \Delta x A^2$$



In Δt, $\Delta x = v \Delta t$

$$\boxed{\frac{\Delta E}{\Delta t} = \frac{1}{2}\omega^2 \mu \frac{\Delta x}{\Delta t} A^2 = \frac{1}{2}\omega^2 \mu v A^2 = \text{Power}}$$

Note that energy, $E \propto A^2$, $E \propto \omega^2$

$E \propto \mu$

$$\text{Power} \propto \frac{1}{\Delta t}$$

Longitudinal waves

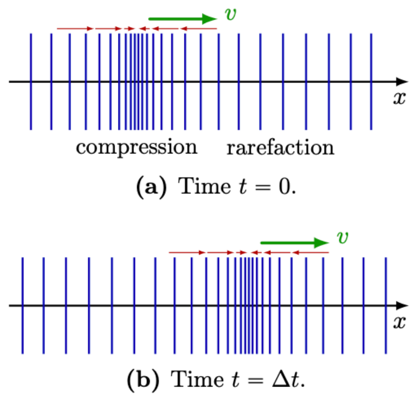
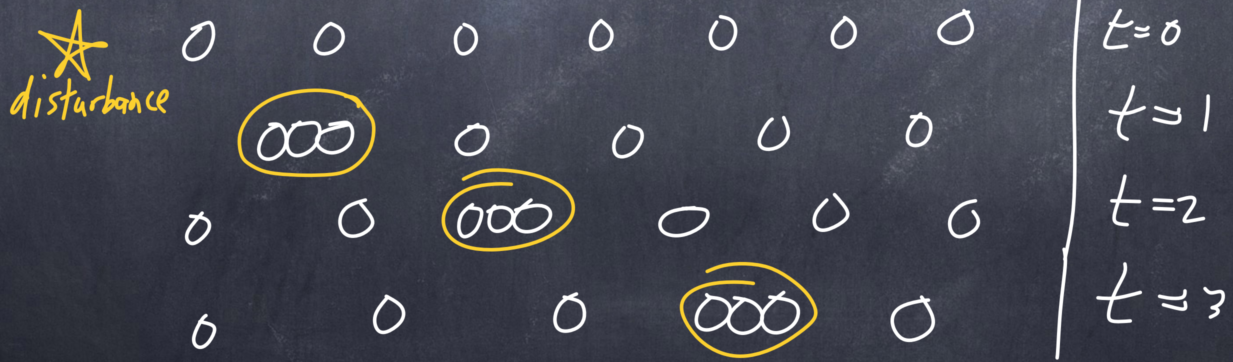


Figure 13.10: A traveling longitudinal wave is when the distortion happens along the direction of propagation, here shown as a local displacement.

Sound waves are longitudinal waves

in air or a fluid.

air molecules



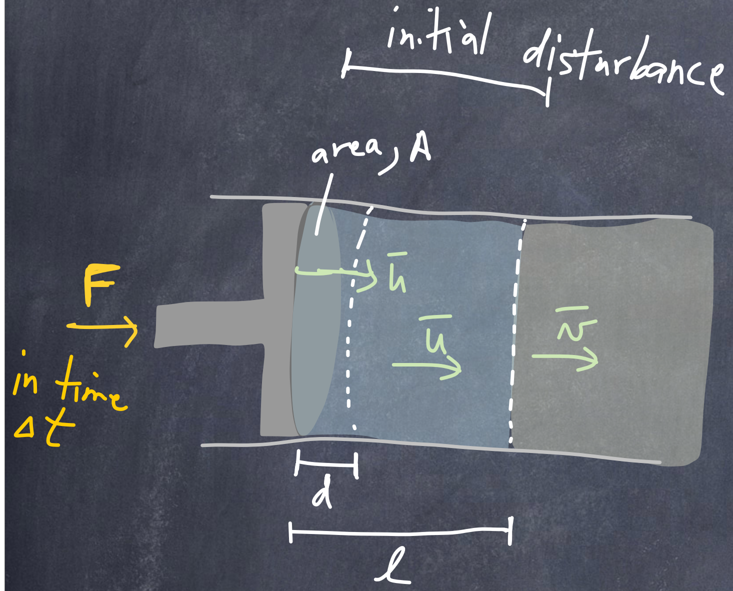
$\longrightarrow$ disturbance propagates what is sound? A pressure increase ΔP that moves with a velocity that depends on medium.

How fast is sound in a fluid?

Derivation: ρ: fluid density

B = Bulk modulus

remember $\boxed{B = \frac{\Delta P}{-\left(\frac{\Delta V}{V}\right)}}$ (1)



we push the piston with velocity u in some time Δt, so a distance $d = u\Delta t$

The fluid up to a distance l is pushed with velocity u. l is defined by the speed of the wave v, where $v \gg u$

The disturbance propagates with velocity, v.

1) compress our piston quickly in time Δt, with velocity $u \Rightarrow d = u\Delta t$

This causes a pressure increase $\Delta P = \frac{F}{A}$

$\boxed{F = A\Delta P}$ ③

2) This creates a pulse of pressure, which is a sound wave $\equiv$ pressure wave.

The wave has velocity v

it moves a distance $l = v\Delta t$

The mass of the fluid that moves is $m = \rho V$

$\boxed{m = \rho A \cdot l = \rho A v \Delta t}$ ②

③ we want to know v. we can do this with momentum conservation.

we can go from $F \rightarrow$ impulse, $\Delta p \rightarrow$ momentum, $p \rightarrow$ velocity

using ③ Impulse $= F\Delta t = A\Delta P \Delta t$ ④

p = momentum $\neq P$ = pressure

Momentum conservation tells us that:

Δp of piston $= \Delta p$ of fluid

from ④ $A\Delta P\Delta t = m\bar{u} = \underbrace{\rho A v \Delta t}_{\text{from ②}}\,\bar{u}$

$$\cancel{A}\,\Delta P\,\cancel{\Delta t} = \rho \cancel{A} v \cancel{\Delta t}\, u$$

$$\Delta P = \rho v u$$

using (1)

$$-B\frac{\Delta V}{V} = \rho v u$$

V: is volume of fluid
ΔV: is volume of fluid moved by piston, $\Delta V = -A\underbrace{u\Delta t}_{d}$ (−) gets compressed.

$$\frac{\Delta V}{V} = \frac{-\cancel{A}u\cancel{\Delta t}}{\cancel{A}v\cancel{\Delta t}} = -\frac{u}{v}$$

$$-B\left(\frac{-u}{v}\right) = \rho v u \qquad \frac{B}{v} = \rho v$$

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$

This is the speed of sound in a fluid. Related to the compressibility + density of the fluid.

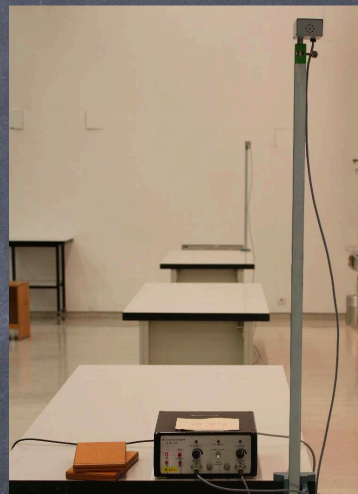
For air, $B = 142 \text{ kPa}$, $\rho = 1.2 \text{ kg/m}^3$

$$v_{\substack{\text{sound}\\ \text{in air}}} = \sqrt{\frac{B}{\rho}} = 343 \ \frac{m}{s}$$

prediction

Note on temperature;
for molecules
$T = \frac{1}{2} m v^2$
$v \propto \sqrt{\text{Temperature}}$

$\Delta x = 7.5 m$



time professor: $\underline{2.5 s}$ $\Rightarrow v_{prof} = \frac{7.5 m}{2.5 s} = 3 m/s$

time student: 23 ms

time sound: 22.5 ms

$$v_{\substack{\text{air}\\ \text{sound}}} = \frac{\Delta x}{\Delta t} = \frac{7.5 m}{22.5 \ e{-3} \ s} = 333 \ \frac{m}{s}$$

Speed of sound in a solid?

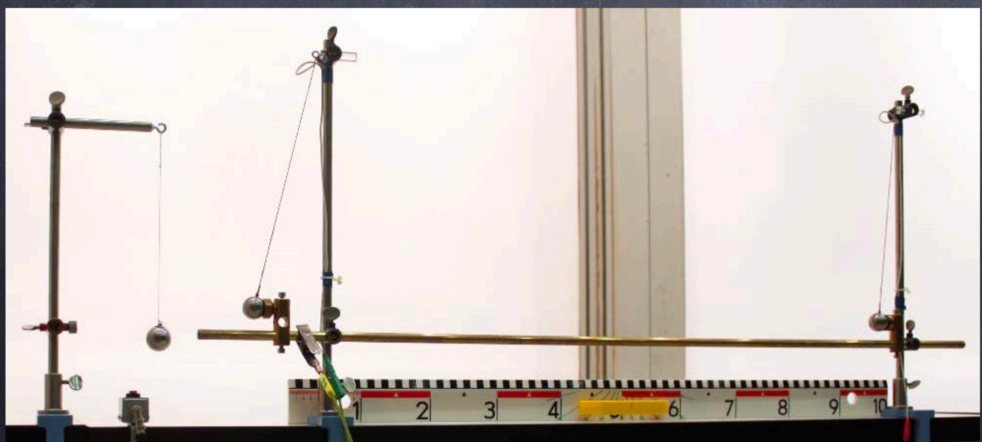
$$v = \sqrt{\frac{Y}{\rho}}$$

Y = Young's module

brass : $Y = 10 \times 10^{10} \frac{N}{m^2}$

$\rho = 8.73 \frac{g}{cm^3} = 8730 \frac{kg}{m^3}$

$$v = \sqrt{\frac{10E10 \frac{N}{m^2}}{8730 \frac{kg}{m^3}}} = 3384 \frac{m}{s}$$ speed of sound in brass



$x = 1\,m$

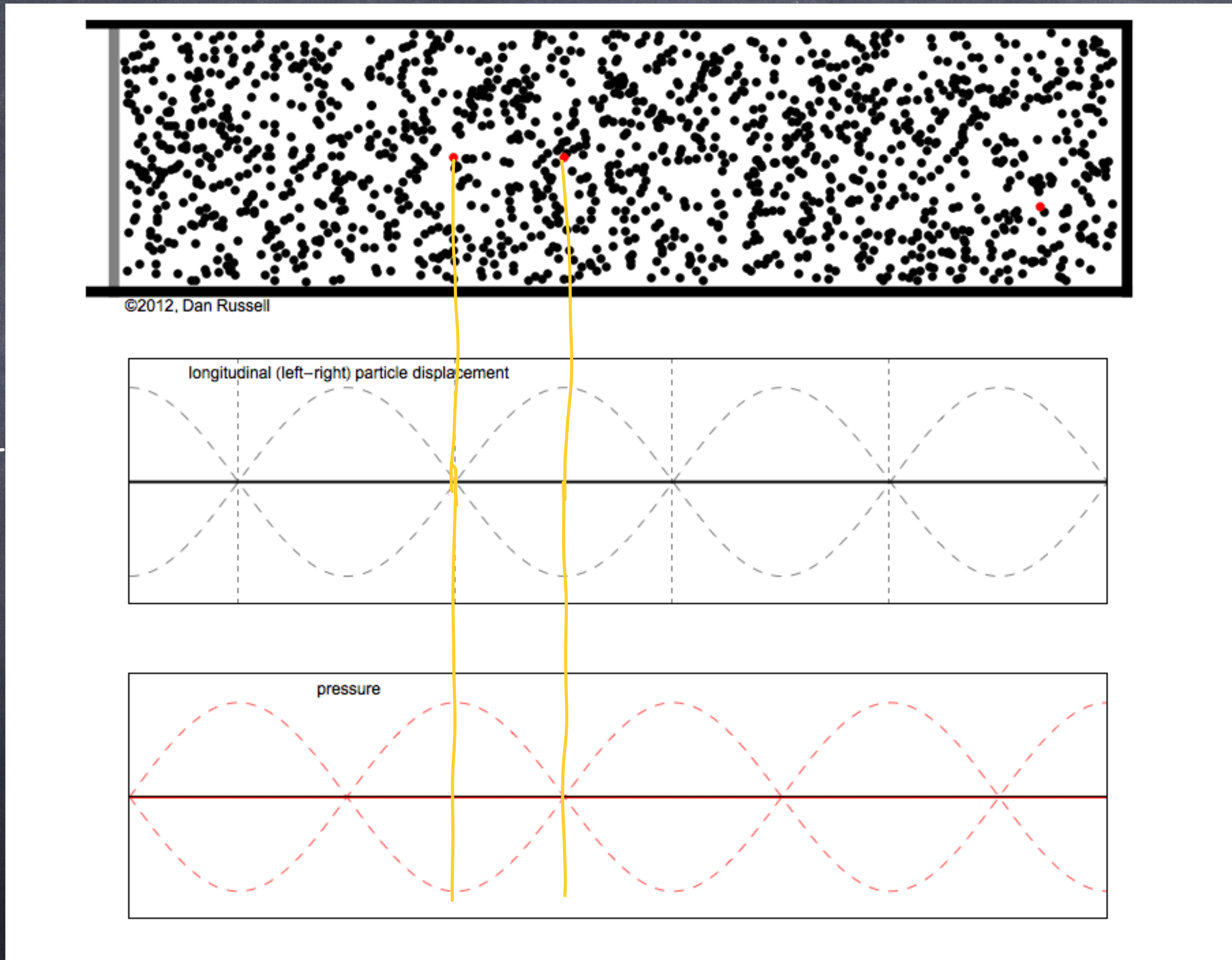
$t = 0.3\ ms$

$$v = \frac{1m}{0,3\ ms} = \frac{1m}{0.3E{-3}\ s}$$

$$v_{\text{sound in brass}} = 3333 \frac{m}{s}$$

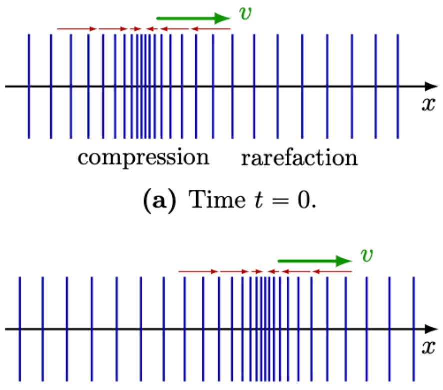
we can create standing waves of sound in a tube

Here, with a piston:



Displacement

Pressure
90° out of phase



(a) Time $t = 0$.

(b) Time $t = \Delta t$.

Figure 13.10: A traveling longitudinal wave is when the distortion happens along the direction of propagation, here shown as a local displacement.

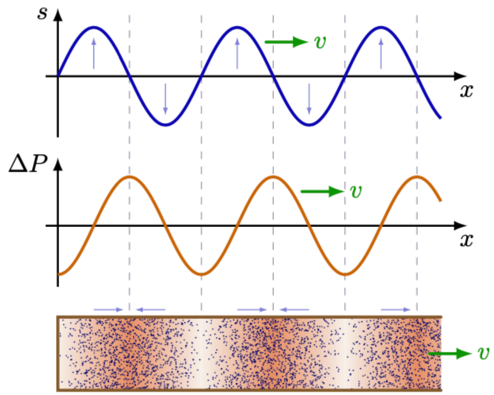


Figure 13.11: Sound wave traveling in a tube of air, shown as a local, average displacement s of air molecules in the longitudinal (x) direction (blue), and a local pressure variation ΔP (orange), 90° out of phase with s.

Displacement $\quad S = S_0 \sin(kx - \omega t)$

maximal displacement

90° phase difference

ΔP = pressure change $\equiv P = P_0 \sin\left(kx - \omega t - \frac{\pi}{2}\right)$

$P_0 = \rho \omega v S_0$

ρ: density of medium

ω: angular frequency

v: speed of sound

Important for today

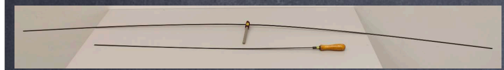
node: position where <u>pressure</u> does <u>not</u> change

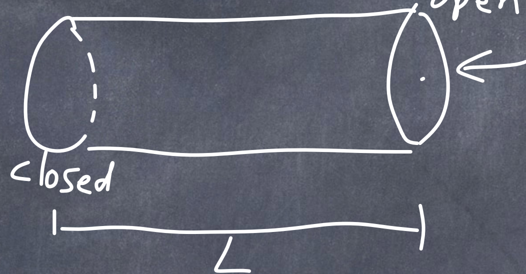
anti-node: position where <u>pressure</u> changes maximally

oscillates between $+\underline{p}$ to $-\underline{p}$

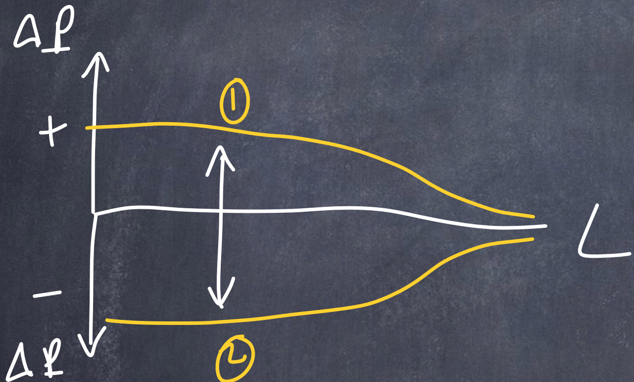
are Δp

Standing waves in a tube closed on one end, open on the other end.

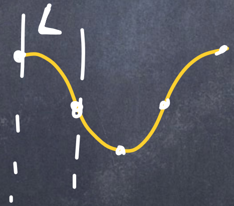




P_{atm} = boundary condition that forces the open end to always have P_{atm} (this is a pressure node)



For $n=1$



$\frac{1}{4}\lambda_1 = L \qquad 4L = \lambda_1 \Rightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$

For $n=3$

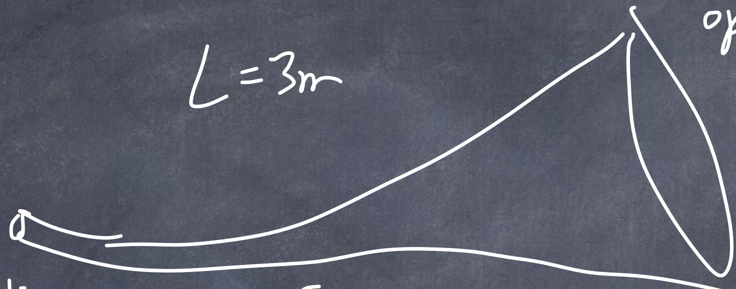
$\frac{3}{4}\lambda_3 = L \qquad \frac{4L}{3} = \lambda_3 \Rightarrow f_3 = \frac{v}{\lambda_3} = \frac{3}{4}\frac{v}{L}$

In general $\quad \lambda_n = \frac{4L}{n} \quad f_n = \frac{v}{\lambda_n} \quad n = 1, 3, 5, \ldots$

Alphorn

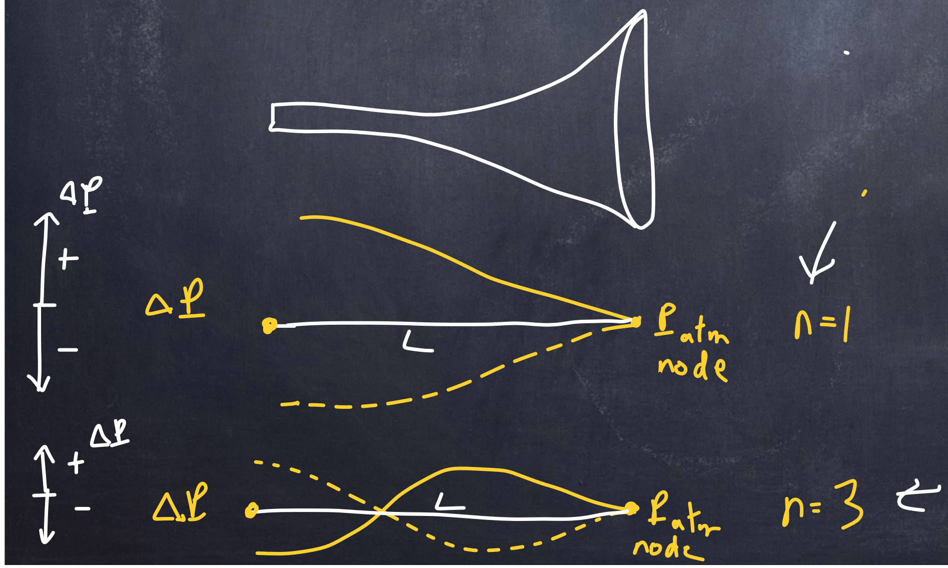
closed end
p changes
(vibrates on lips)
anti-node

$L = 3m$



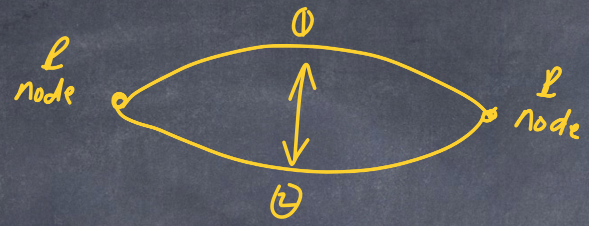
open end = node
$p = p_{atm}$

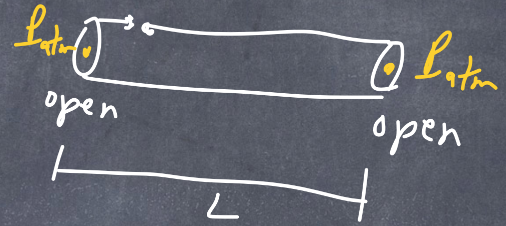
fundamental frequency for Alphorn $= f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} = \frac{340\,m/s}{4(3m)} =$
28 Hz



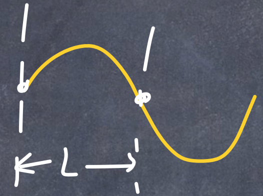
Standing sound waves, tube open on both ends (Flute)

For $n=1$, length, L



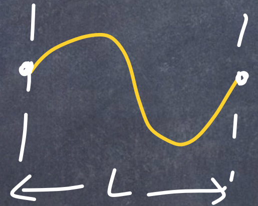


$n=1$



$L = \frac{\lambda_1}{2}$

$n=2$

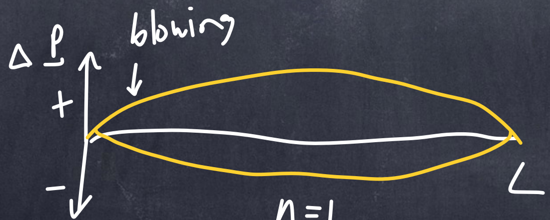


$L = \lambda_2$

In general,

$\lambda_n = \frac{2L}{n}$ $\quad n = 1, 2, \ldots$

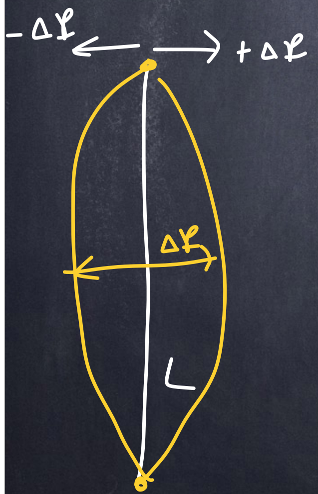
$f_n = \frac{v}{\lambda_n}$ $\quad v = v_{\text{sound in air}}$

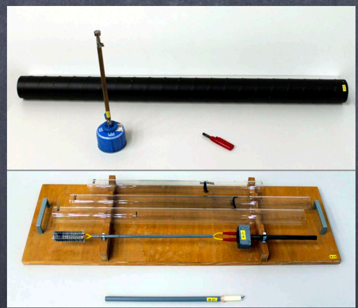


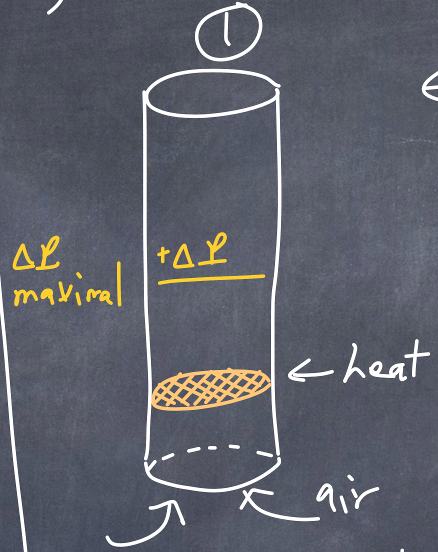
Flute

Finger positions excite different harmonics
(Fingers put anti-nodes)

Rijke tube - self-amplifying standing sound waves.







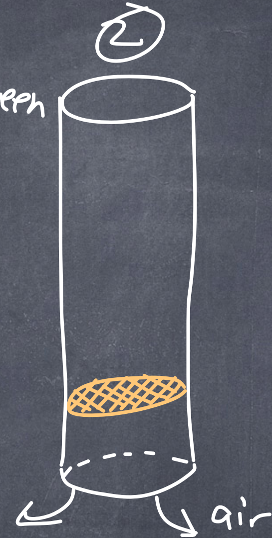
$\frac{1}{2}$ of the cycle,

air come in and create a high pressure region in center

oscillation between ① + ②

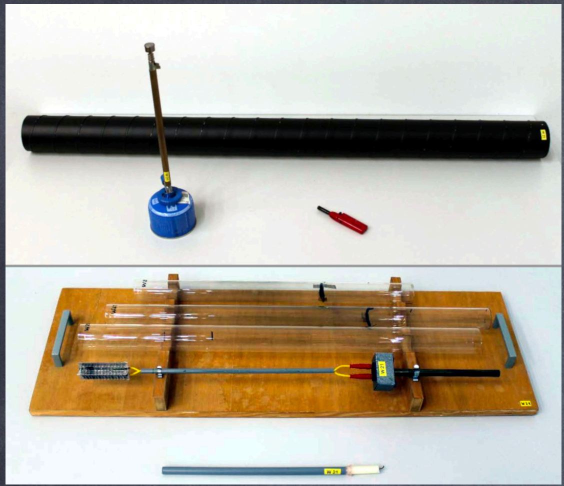
$pV = nRT$

$\Delta p \sim \Delta T$



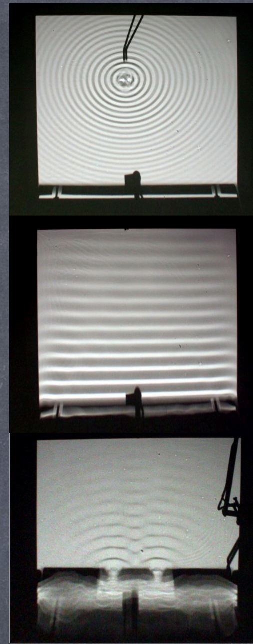
$\frac{1}{2}$ of cycle,

air flows away from high pressure region, away from center, low pressure in center



W21

W13



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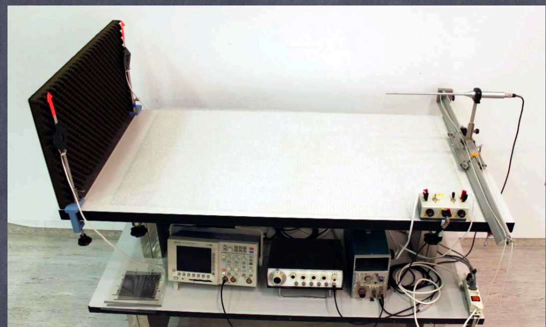
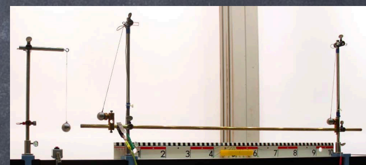


W110

W34



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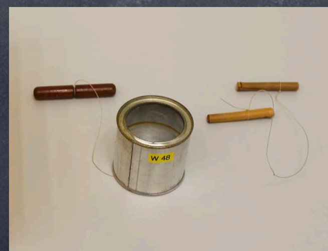

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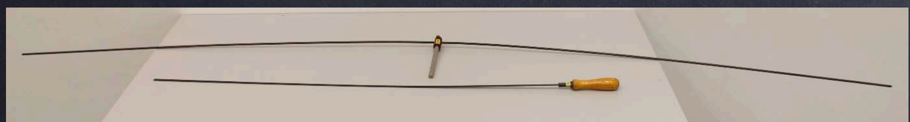
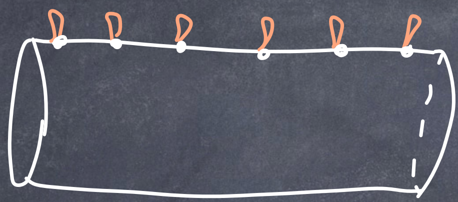


W36

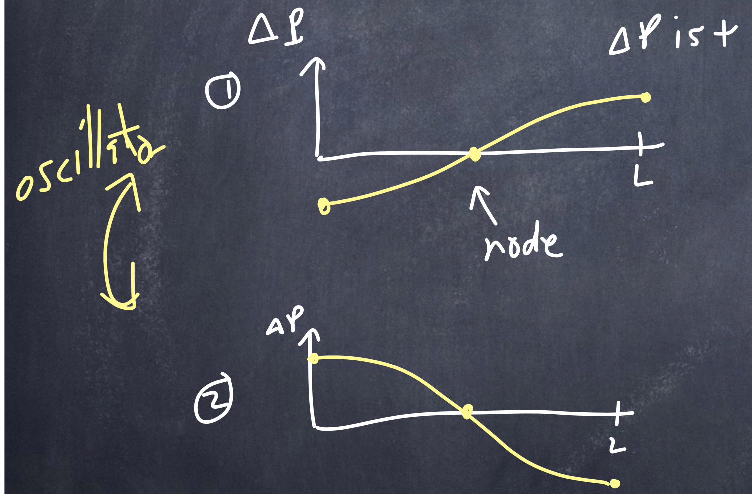
W33

Ruben's flame tube

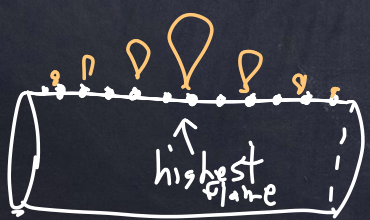
①



n=1



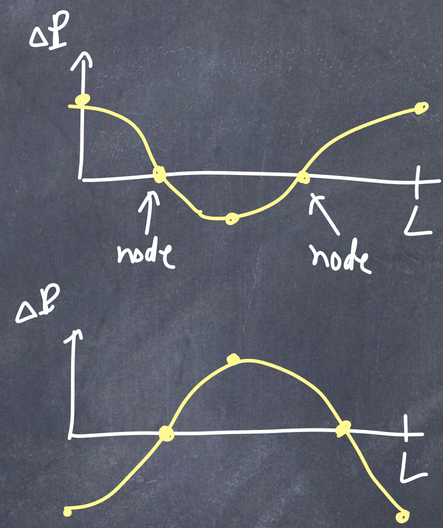
①→②→①...



flame is higher at node because ΔP is smaller than P_{atm}

(see next page)

n=2



For quiet sounds, ΔP of the gas < P of the gas

From Bernoulli's equation, gas flow is proportional to square root of the pressure difference between inside + outside tube.

$$\text{flow} \sim \sqrt{P_{inside} - P_{outside}}$$

(The flow of gas out of the pipe)

$\Delta P_{maximal}$, anti-nodes produce lower flames (flow rate is lower)

$\Delta P = 0$, nodes, flow rate is higher

Part of the cycle, pressure is higher than average but part is lower. On average

This is why pressure is higher at nodes:

$$\sqrt{\text{Pressure difference at anti-nodes}} < \sqrt{\text{Pressure}} \text{ at nodes.}$$